Robust fuzzy control of electrical drive

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Abstract

Mamdani and Takagi-Sugeno fuzzy controllers were prepared. The controllers were designed using Fuzzy Logic Toolbox. The membership functions and rules were used as design tools that give opportunity to model a control surface and controller properties.

The control system of d.c. (direct current) motor drive with two feedback loops is considered. First loop is used for stabilization of armature current (proportional to electrical torque), second loop is used for speed stabilization. Simulink model was used for simulation of d.c. motor drive and mechanical system. The moment of inertia and disturbance load were changed independently several times.

Time response of armature current and motor speed against disturbances of load torque and moment of inertia was tested for the above system. Drive systems using fuzzy controllers (Mamdani and Sugeno) were compared with drive using conventional PI controllers. Nonlinear Control Design (NCD) Blockset (an extension of Simulink) was used for tuning PI controller parameters.

Keywords: fuzzy logic control, Mamdani-type fuzzy system, Sugeno-type fuzzy system, d.c. drive

1. Introduction

The conventional control design is based on a mathematical model of a plant. Real plant is always nonlinear. Electrical parameters of d.c. motor are nonlinear due to saturation and to temperature variations. Gain of converter is nonlinear too. Mechanical load parameters and moment of inertia may vary within limits, which are known or determined separately.

In a fuzzy control system there are many possibilities for adaptation to large unknown variation of plant parameters, without using a mathematical model. This is discussed below.

2. Simulation model of electrical drive

A linear model of d.c. drive will be used. The model of electrical drive consists of following parts: electrical motor with load (mechanical system) and the thyristor converter, which supplies the d.c. motor. A model of d.c. drive with classical control system was built using Simulink blocks (Fig. 2). Its parameters are computed by custom block Run inidcdr22kW (shown in Fig.2) from rated catalogue data: motor power 22[kW], supplied voltage 440[V], current 56.2[A], speed 1500 r.p.m., etc.

2.1. Model of d.c. motor and mechanical load

A linear model of d.c. motor was built in Simulink as custom subsystem (MotorDC_I block shown in Fig. 2) [3]. It assumed that the d.c. machine is separately exited with constant field current. The speed may be controlled from zero up to the rated value. The driven mechanism can be considered as a linear system. The system equations are then

\[
L_a \frac{dI_a}{dt} = -R_a I_a - k_a \omega_a + U_{sa}(t)
\]

\[
J_{obl} \frac{d\omega_a}{dt} = k_m I_a - M_{st}(t)
\]

where:

- \(I_a\), \(U_{sa}\) - armature current and supplied voltage,
- \(R_a, L_a, k_a\) - armature resistance, inductance, d.c. motor constant,
- \(J_{obl}, M_{st}, \omega_a\) - total inertia, load torque, angular motor velocity.

There are two inputs (armature supplied voltage and load torque) and two outputs (angular velocity and armature current).

2.2. Model of converter/rectifier

The three-phase bridge converter is often used in motor-control systems. Two of six thyristors conduct at any time instant. Gating of each thyristor initiates a pulse of load current; therefore this is a six-pulse controlled rectifier (converter).

The dead time may vary from zero to one-six the period of an a.c. source (0 ÷ 3.33 ms for 50 Hz). It is assumed that the mean dead time is \(T_{mip}=1.67\)[ms]. The model of converter is described as first order inertia with transfer function [6]:

\[
G_{conv}(s) = \frac{U_{sa}(s)}{u_s(s)} = \frac{k_p}{T_{mip} \cdot s + 1}
\]

where:

- \(k_p\) - voltage amplification of converter (rectifier),
- \(T_{mip}\) - mean dead time of converter (rectifier)

The output voltage of converter is [6]:

\[
U_{sa} = U_{s0} \cos \alpha = U_{s0} \cos \left( \frac{\pi}{2} \left( 1 - \frac{u_{st}}{u_{max}} \right) \right)
\]

where:

- \(U_{s0}\), \(\alpha\) - ideal no-load d.c. voltage, delay angle,
- \(u_{st}\), \(u_{max}\) - control voltage of converter.

The voltage amplification of converter is non-linear, as shown in Fig. 1. The output voltage of armature current controller (\(u_{sa}\)) is used to control the firing angle of thyristors in the three-phase full-wave converter which supplies the d.c. motor.

![Fig.1 Output voltage and voltage amplification versus control voltage of converter.](image-url)
2.3. Control system of d.c. motor drive

The control system of d.c. motor drive with two feedback loops is considered. First loop is used for stabilization of the armature current (electrical torque). Second loop is used for stabilization speed of d.c. motor. The control system of d.c. motor drive with two classical PI controllers is presented in Fig. 2. The armature current error is the input signal to the armature controller, and it is the difference of the reference armature current and the actual armature current (ia – iaz), where the maximum value of armature current is limited by setting of reference armature current value.

The speed controller and the armature current controller form a cascade control system. The difference of reference speed and actual speed (ns – n) produces the speed error, which acts as an input signal to the speed controller. An output of the speed controller is the reference signal of the armature current controller, iaz. It should be noted that each controller has been limited, and the armature current is well controlled.

3. Tuning parameters of classical PI controllers

The Nonlinear Control Design Blockset (NCD) was used for tuning of the controllers’ parameters, to inspect the design criteria for system with a unit step input. The tuning parameters for current controller and it is the difference of the reference armature current and the actual armature current (ia – iaz), where the maximum value of armature current is limited by setting of reference armature current value.

The speed controller and the armature current controller form a cascade control system. The difference of reference speed and actual speed (ns – n) produces the speed error, which acts as an input signal to the speed controller. An output of the speed controller is the reference signal of the armature current controller, iaz. It should be noted that each controller has been limited, and the armature current is well controlled.

3.1. Tuning parameters for current controller

A plant model in current loop circuit was build as series connections of transfer functions of the converter and of armature-circuit current, as shown in Fig. 3.

The transfer function of the armature-circuit current is:

$$G_a(s) = \frac{i_a(s)}{U_{ia}(s)} = \frac{k_{ia}}{T_a \cdot s + 1}$$

where: $k_{ia}$ - gain of armature current circuit, $T_a = L_a/R_a$ - armature-circuit time constant.

3.2. Tuning parameters for speed controller

A plant model in speed loop circuit was build as series connections of transfer functions of the already designed current loop and of the mechanical system, as shown in Fig. 5. The transfer function of the motor and its load (driven mechanism) is:

$$G_m(s) = \frac{\omega_m(s)}{i_a(s)} = \frac{k_{om} R_s}{k_m} \cdot \frac{1}{T_m \cdot s}$$

where: $k_{om}$, $R_s$ - gain of speed circuit, total circuit resistance, $T_m = J_{el}R_s/k_m^2$ - electromechanical time constant.

The rise time, setting time and the overshoot of current were adjusted in NCD Blockset constraint window for the step time input, as shown in Fig. 4. Two uncertain variables were incorporated into the optimisation [3]. There are armature-circuit time constant $T_a$ and voltage amplification of converter $kp$. Their ranges are: $T_a$ varies 10% and $kp$ varies between 34 and 101.
The rise time, setting time and overshoot of motor speed for the step time response are adjusted in NCD constraint window (Fig. 6). The uncertain variables and their ranges are: total circuit resistance \( R_c \) varies 10\%, electromechanical time constant \( T_m \) varies between 0.58[s] and 1.71[s] when total inertia \( J_{tot} \) varies respectively between 2.7[kgm²] and 6[kgm²].

When tuning with NCD Blockset is finished, the transient response is within limits adjusted in NCD window. The transient was initially well outside the adjusted constraints.

4. DC drive with fuzzy speed and armature current controller

Fuzzy controllers replace the speed and armature current controllers shown in Fig.2 [7]. In the first type of implementation, both the speed and armature current controllers are Mamdani-type PI fuzzy logic controllers [2]. In the second implementation, two fuzzy logic Sugeno-type PI controllers are used.

Both fuzzy controllers: Mamdani- and Sugeno-type, are implemented with the Fuzzy Logic Toolbox. It provides a graphical user interface (FIS Editor) [1], [3] for defining the fuzzy controller. The controller fuzzy designed may be hidden in a fuzzy block to be included in Simulink block diagram (Fig. 9).

4.1. Fuzzy controllers as the Mamdani system

The fuzzy logic controllers of Mamdani-type use two fuzzy variables \( E(\text{error}) \) and \( CE(\text{change of error}) \) as input, and one output fuzzy variable \( CU(\text{change of output}) \). The linguistic rules of the fuzzy logic controller take the form:

If \( E = A \) and \( CE = B \) then \( CU = C \)

where: \( A, B \) and \( C \) denote fuzzy set as linguistic variables (e.g. Small Positive, etc.)

Seven linguistic variables (from Big Negative to Big Positive) are used. The belongingness to fuzzy set is described by the membership functions. The input and output membership functions are simple Gaussian curves. There are altogether 22 rules considered. The same rules are used for both the speed and the current fuzzy controllers. The input/output block diagram of a fuzzy controller and the membership functions for fuzzy variable \( error \) are shown in Fig. 7.

The application of fuzzy PI-type controllers (for armature current and speed) give satisfactory performance when Gaussian membership functions are appropriately tuned [2].

Membership functions and rule base are design tools that give opportunity to model a control surface and controller properties. It is obvious that using this attributes one can more precisely fulfil a quality criterion in full operational range.

4.2. Fuzzy controllers as the Sugeno system

In the Sugeno-type fuzzy system, the rules take such a form that the antecedent is a function of the input variables, i.e. a typical rule in the Sugeno-type fuzzy system with two inputs is

If \( x \) is \( A \) and \( y \) is \( B \) then \( z = f(x,y) \)

where: \( x \) and \( y \) are input variables, \( A \) and \( B \) are fuzzy sets, and \( f(x,y) \) is a function and \( z \) is the crisp output. When the function \( f(x,y) \) is a first-order polynomial, the resulting fuzzy interference system is called a first-order Sugeno fuzzy model. In this case, in general, an \( i \)-th rule results in an output:

\[
 z_i = f(x,y) = p_i x + q_i y + r_i
\]

where: \( p_i, q_i, \) and \( r_i \) are constant.

The armature current fuzzy controller of Sugeno-type uses one fuzzy variable \( U_a \) (supplied voltage), crisp set \( prop \) (current error) and crisp set \( int \) (for change of current error) as inputs. One output fuzzy variable \( u_a \) (change of output) is used. This tree-inputs-single-output system (Fig. 9) has only tree rules with tree linguistic input/output variables:

If \( U_a \) is small then \( u_a = k_{a1} \cdot prop + t_{a1} \cdot int \)
If \( U_a \) is medium then \( u_a = k_{a2} \cdot prop + t_{a2} \cdot int \)
If \( U_a \) is large then \( u_a = k_{a3} \cdot prop + t_{a3} \cdot int \)

where: \( k_{a1}, t_{a1}, k_{a2}, t_{a2}, k_{a3}, t_{a3} \) are proportional and integral gains for classical current PI controller. All gains are tuned with NCD Blockset respectively for the voltage amplification \( kp \in [101 \ 93] \) (small), \( kp \in [93 \ 71] \) (medium), \( kp \in [71 \ 34] \) (large).

The speed motor fuzzy controller of Sugeno-type uses one fuzzy variable \( J_{tot} \) (total inertia), crisp set \( prop \) (for speed error) and crisp set \( int \) (for change of speed error) as inputs. One output fuzzy variable \( \omega_a \) (change of output) is used. This tree-inputs-single-output system (Fig. 9) has four rules with four linguistic input/output variables:

If \( J_{tot} \) is small then \( \omega_a = k_{o1} \cdot prop + t_{o1} \cdot int \)
If \( J_{tot} \) is nominal then \( \omega_a = k_{o2} \cdot prop + t_{o2} \cdot int \)
If \( J_{tot} \) is medium then \( \omega_a = k_{o3} \cdot prop + t_{o3} \cdot int \)
If \( J_{tot} \) is large then \( \omega_a = k_{o4} \cdot prop + t_{o4} \cdot int \)

where: \( k_{o1}, t_{o1}, k_{o2}, t_{o2}, k_{o3}, t_{o3}, k_{o4}, t_{o4} \) are proportional and integral gains for classical speed PI controller. All gains are tuned with NCD Blockset for total inertia of the motor and the mechanism coupled to it respectively \( J_{tot} \in [1.6 \ 2.7] \) (small), \( J_{tot} \in [2.0 \ 2.7] \) (nominal), \( J_{tot} \in [2.7 \ 4.7] \) (medium), \( J_{tot} \in [4.7 \ 6.0] \) (large).

Simple Gaussian curves are used as input and output membership functions for both designed fuzzy Sugeno-type controllers.
5. Simulation results

The start of d.c. drive from \( n_0 = 0 \) to the reference speed \( n_r = 114 \text{[rad/s]} \) (e.g. 0.75\( n_N \)), with load torque equal the half of rated value \( (M_L = 0.5M_{\text{in}}) \) was chosen for tests. At time \( t=1.5\text{s} \), the load torque was step increased to rated value \( (M_L = M_{\text{in}}) \). This simulation was done for two values of total inertia of the mechanical system: \( J_{\text{in}} = 2.7 \text{[kgm}^2\] \) and \( J_{\text{in}} = 6 \text{[kgm}^2\] \).

Figure 10 shows the current and speed time responses for the drive scheme using two classical PI controllers (current and speed), as shown in Fig. 2.

![Classical control system of d.c. drive](image)

**Fig. 10.** Time responses for d.c. drive with two classical PI controllers

Figure 11 shows the current and speed time responses in the drive scheme using two fuzzy Sugeno-type PI controllers (current and speed), as shown in Fig. 9.

![Fuzzy control system of d.c. drive](image)

**Fig. 11.** Time responses for d.c. drive with two Sugeno-type fuzzy PI controllers

6. Final remarks

Tuning of the fuzzy Sugeno-type PI controller is faster and easier than the fuzzy Mamdani-type PI controller. It is a great advantage of the fuzzy Sugeno-type controller that it gives the possibility to apply classical control design for the fuzzy controller.

The MATLAB/Simulink with NCD Blockset and Fuzzy Logic Toolbox is an effective and user-friendly tool for designing and testing of a fuzzy controller of Mamdani- and Sugeno-type.

References

[7] P. Vas Artificial Intelligence Based Electrical Machines and Drives, OXFORD University Press, 1999